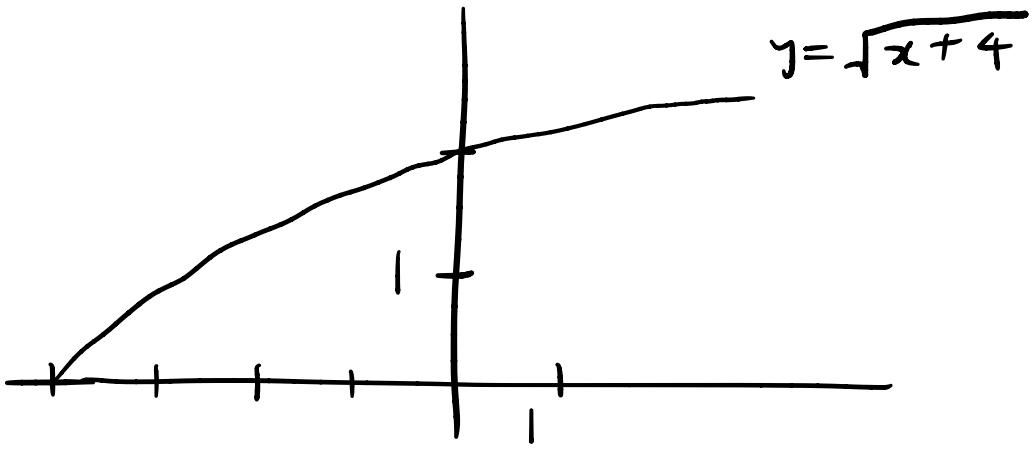
What point on the curve $y=\sqrt{x+4}$ comes closest to the origin?



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$$y = \sqrt{x+4}$$

$$\text{distance} = \sqrt{(y-y_0)^2+(x-x_0)^2}, \quad \text{origin} = (0,0)$$

$$\Rightarrow f(x) = \sqrt{\sqrt{x+4}^2 + x^2}$$

$$= \sqrt{x^2+x+4}$$

$$f'(x) = \frac{1}{2}(x^2+x+4)^{-\frac{1}{2}} \cdot (2x+1) = -\frac{1 \pm \sqrt{-15}}{2}$$

$$= \frac{2x+1}{2\sqrt{x^2+x+4}}$$

$$f'(x) = 0 \Rightarrow 2x+1 = 0 \Rightarrow x = -\frac{1}{2}$$

At $x = -4$, $y = 0$, $f(x) = 4$ $\quad \therefore \left(-\frac{1}{2}, \frac{\sqrt{14}}{2}\right)$

at $x = 0$, $y = 2$, $f(x) = 2$

at $x = -\frac{1}{2}$, $y = \sqrt{\frac{7}{2}}$, $f(x) = \frac{\sqrt{15}}{2}$